CHAPTER – 6

SEQUENCE AND SERIES- ARITHMETIC AND GEOMETRIC PROGRESSIONS

LEARNING OBJECTIVES

Often students will come across a sequence of numbers which are having a common difference, i.e., difference between the two consecutive pairs are the same. Also another very common sequence of numbers which are having common ratio, i.e., ratio of two consecutive pairs are the same. Could you guess what these special type of sequences are termed in mathematics?

Read this chapter to understand that these two special type of sequences are called Arithmetic Progression and Geometric Progression respectively. Further learn how to find out an element of these special sequences and how to find sum of these sequences.

These sequences will be useful for understanding various formulae of accounting and finance.

The topics of sequence, series, A.P., G.P. find useful applications in commercial problems among others; viz., to find interest earned on compound interest, depreciations after certain amount of time and total sum on recurring deposits, etc.

6.1 SEQUENCE

Let us consider the following collection of numbers-

(1) 28 , 2, 25, 27, ——————

(2) 2 , 7, 11, 19, 31, 51, ——————

(3) 1, 2, 3, 4, 5, 6, ——————

(4) 20, 18, 16, 14, 12, 10, ——————

In (1) the nos. are not arranged in a particular order. In (2) the nos. are in ascending order but they do not obey any rule or law. It is, therefore, not possible to indicate the number next to 51.

In (3) we find that by adding 1 to any number, we get the next one. Here the no. next to 6 is (6 + 1 =) 7.

In (4) if we subtract 2 from any no. we get the nos. that follows. Here the no. next to 10 is (10 –2 =) 8.

Under these circumstances, we say, the nos. in the collections (1) and (2) do not form sequences whereas the nos. in the collections (3) & (4) form sequences.

Thus a sequence may be defined as follows:—

An ordered collection of numbers a_1, a_2, a_3, a_4, ..., a_n, ... is a sequence if according to some definite rule or law, there is a definite value of a_n, called the term or element of the sequence, corresponding to any value of the natural no. n.

Clearly, a_1 is the 1st term of the sequence , a_2 is the 2nd term, ..., a_n is the nth term.

In the nth term a_n, by putting n = 1, 2 ,3 ,......... successively , we get a_1, a_2 , a_3 , a_4, ...

Thus it is clear that the nth term of a sequence is a function of the positive integer n. The nth term is also called the general term of the sequence. To specify a sequence, nth term must be known, otherwise it may lead to confusion. A sequence may be finite or infinite.

If the number of elements in a sequence is finite, the sequence is called *finite sequence*; while if the number of elements is unending, the sequence is *infinite*.

A finite sequence a_1, a_2, a_3, a_4, ..., a_n is denoted by $\{a_i\}_{i=1}^{n}$ and an infinite sequence a_1, a_2, a_3, a_4, ..., a_n ,................. is denoted by $\{a_n\}_{n=1}^{\infty}$ or simply by $\{a_n\}$ where a_n is the nth element of the sequence.

Example :

1) The sequence { 1/n } is 1, $^1/_2$, $^1/_3$, $^1/_4$,……
2) The sequence { $(-1)^n$ n } is –1, 2, –3, 4, –5,…..
3) The sequence { n } is 1, 2, 3,…
4) The sequence { n / (n + 1) } is $^1/_2$, $^2/_3$, $^3/_4$, $^4/_5$, …….
5) A sequence of even positive integers is 2, 4, 6, ...
6) A sequence of odd positive integers is 1, 3, 5, 7, ...

All the above are infinite sequences.

Example:

1) A sequence of even positive integers within 12 i.e., is 2, 4, 6, 10.
2) A sequence of odd positive integers within 11 i.e., is 1, 3, 5, 7, 9. etc.

All the above are finite sequences.

6.2 SERIES

An expression of the form $a_1 + a_2 + a_3 + \ldots.. + a_n + \ldots\ldots\ldots$ which is the sum of the elements of the sequenece { a_n } is called a *series*. If the series contains a finite number of elements, it is called a *finite series*, otherwise called *an infinite series*.

If $S_n = u_1 + u_2 + u_3 + u_4 + \ldots\ldots + u_n$, then S_n is called the sum to n terms (or the sum of the first n terms) of the series and is denoted by the Greek letter sigma Σ.

Thus, $S_n = \sum_{r=1}^{n} u_r$ or simply by Σu_n.

Illustrations :

(i) 1 + 3 + 5 + 7 + ... is a series in which 1st term = 1, 2nd term = 3 , and so on.

(ii) 2 – 4 + 8 –16 + ... is also a series in which 1st term = 2, 2nd term = –4 , and so on.

6.3 ARITHMETIC PROGRESSION (A.P.)

A sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ is called an Arithmetic Progression (A.P.) when $a_2 - a_1 = a_3 - a_2 = \ldots.. = a_n - a_{n-1}$. That means A. P. is a sequence in which each term is obtained by adding a constant d to the preceding term. This constant 'd' is called the *common difference* of the A.P. If 3 numbers a, b, c are in A.P., we say

b – a = c – b or a + c = 2b; b is called the arithmetic mean between a and c.

Example: 1) 2,5,8,11,14,17,…… is an A.P. in which d = 3 is the common diference.

2) 15,13,11,9,7,5,3,1,–1, is an A.P. in which –2 is the common difference.

Solution: In (1) 2nd term = 5 , 1st term = 2, 3rd term = 8,

so 2nd term – 1st term = 5 – 2 = 3, 3rd term – 2nd term = 8 – 5 = 3

Here the difference between a term and the preceding term is same that is always constant. This constant is called common difference.

Now in generel an A.P. series can be written as

a, a + d, a + 2d, a + 3d, a + 4d, ……

where 'a' is the 1^{st} term and 'd' is the common difference.

Thus 1^{st} term (t_1) = a = a + (1 – 1) d

2^{nd} term (t_2) = a + d = a + (2 – 1) d

3^{rd} term (t_3) = a + 2d = a + (3 – 1) d

4^{th} term (t_4) = a + 3d = a + (4 – 1) d

…………………………………………….

n^{th} term (t_n) = a + (n – 1) d, where n is the position no. of the term .

Using this formula we can get

50^{th} term (= t_{50}) = a+ (50 – 1) d = a + 49d

Example 1: Find the 7th term of the A.P. 8, 5, 2, –1, –4,…..

Solution : Here a = 8, d = 5 – 8 = –3

Now t_7 = 8 + (7 – 1) d

= 8 + (7 – 1) (– 3)

= 8 + 6 (– 3)

= 8 – 18

= – 10

Example 2 : Which term of the AP $\frac{3}{\sqrt{7}}, \frac{4}{\sqrt{7}}, \frac{5}{\sqrt{7}}$ ....is $\frac{17}{\sqrt{7}}$?

Solution : $a = \frac{3}{\sqrt{7}}$, $d = \frac{4}{\sqrt{7}} - \frac{3}{\sqrt{7}} = \frac{1}{\sqrt{7}}$, $t_n = \frac{17}{\sqrt{7}}$

We may write

$$\frac{17}{\sqrt{7}} = \frac{3}{\sqrt{7}} + (n-1) \times \frac{1}{\sqrt{7}}$$

or, 17 = 3 + (n – 1)

or, $n = 17 - 2 = 15$

Hence, 15^{th} term of the A.P. is $\frac{17}{\sqrt{7}}$.

Example 3: If 5^{th} and 12^{th} terms of an A.P. are 14 and 35 respectively, find the A.P.

Solution: Let a be the 1^{st} term & d be the common difference of A.P.

$t_5 = a + 4d = 14$

$t_{12} = a + 11d = 35$

On solving the above two equations:

$7d = 21$ = i.e., $d = 3$

and $a = 14 - (4 \times 3) = 14 - 12 = 2$

Hence, the required A.P. is 2, 5, 8, 11, 14,……………

Example 4: Divide 69 into three parts which are in A.P. and are such that the product of the 1^{st} two parts is 483.

Solution: Given that the three parts are in A.P., let the three parts which are in A.P. be $a - d$, a, $a + d$.........

Thus $a - d + a + a + d = 69$

or $3a = 69$

or $a = 23$

So the three parts are $23 - d$, 23, $23 + d$

Since the product of first two parts is 483, therefore, we have

$23 (23 - d) = 483$

or $23 - d = 483 / 23 = 21$

or $d = 23 - 21 = 2$

Hence, the three parts which are in A.P. are

$23 - 2 = 21$, 23, $23 + 2 = 25$

Finally the parts are 21, 23, 25.

Example 5: Find the arithmetic mean between 4 and 10.

Solution: We know that the A.M. of a & b is $= (a + b) /2$

Hence, The A. M between 4 & 10 $= (4 + 10) /2 = 7$

Example 6: Insert 4 arithmetic means between 4 and 324.

4, –, –, –, –, 324

Solution: Here a= 4, d = ? n = 2 + 4 = 6, t_n = 324

Now $t_n = a + (n-1)d$

or $324 = 4 + (6-1)d$

or $320 = 5d$ i.e., = i.e., $d = 320 / 5 = 64$

So the 1st AM = 4 + 64 = 68

2nd AM = 68 + 64 = 132

3rd AM = 132 + 64 = 196

4th AM = 196 + 64 = 260

Sum of the first n terms

Let S be the Sum, a be the 1st term and ℓ the last term of an A.P. If the number of term are n, then $t_n = \ell$. Let d be the common difference of the A.P.

Now $S = a + (a+d) + (a+2d) + .. + (\ell - 2d) + (\ell - d) + \ell$

Again $S = \ell + (\ell - d) + (\ell - 2d) + ... + (a+2d) + (a+d) + a$

On adding the above, we have

$2S = (a+\ell) + (a+\ell) + (a+\ell) + ... + (a+\ell)$

$= n(a+\ell)$

or $S = n(a+\ell)/2$

Note: The above formula may be used to determine the sum of n terms of an A.P. when the first term a and the last term is given.

Now $l = t_n = a + (n-1)d$

$$\therefore \ S = \frac{n\{a + a + (n-1)d\}}{2}$$

or $$s = \frac{n}{2}\{2a + (n-1)d\}$$

Note: The above formula may be used when the first term a, common difference d and the number of terms of an A.P. are given.

Sum of 1st n natural or counting numbers

$S = 1 + 2 + 3 + ... + ... (n-2) + (n-1) + n$

Again $S = n + (n-1) + (n-2) + ... + 3 + 2 + 1$

On adding the above, we get

$2S = (n+1) + (n+1) + ...$ to n terms

or $2S = n(n+1)$

$S = n(n+1)/2$

Then Sum of 1^{st}, n natural number is n(n + 1) / 2

i.e. $1 + 2 + 3 + \dots\dots + n = \dfrac{n(n+1)}{2}$.

Sum of 1^{st} n odd number

$S = 1 + 3 + 5 + \dots\dots + (2n - 1)$

Sum of 1^{st} n odd number

$S = 1 + 3 + 5 + \dots\dots + (2n - 1)$

Since $S = n\{ 2a + (n - 1) d \} / 2$, we find

$$S = \frac{n}{2}\{ 2.1 + (n - 1) 2 \} = \frac{n}{2}(2n)\, n^2$$

or $S = n^2$

Then sum of 1^{st}, n odd numbers is n^2, i.e. $1 + 3 + 5 + \dots + (2n - 1) = n^2$

Sum of the Squares of the 1^{st}, n natural nos.

Let $S = 1^2 + 2^2 + 3^2 + \dots\dots + n^2$

We know $m^3 - (m - 1)^3 = 3m^2 - 3m + 1$

We put $m = 1, 2, 3, \dots\dots, n$

$$1^3 - 0 = 3.1^2 - 3.1 + 1$$
$$2^3 - 1^3 = 3.2^2 - 3.2 + 1$$
$$3^3 - 2^3 = 3.3^2 - 3.3 + 1$$
$$\dots\dots\dots\dots\dots\dots$$
$$+\; n^3 - (n - 1)^3 = 3n^2 - 3.n + 1$$

Adding both sides term by term,

$n^3 = 3S - 3\,n (n + 1) / 2 + n$

or $2n^3 = 6S - 3n^2 - 3n + 2n$

or $6S = 2n^3 + 3n^2 + n$

or $6S = n (2n^2 + 3n + 1)$

or $6S = n (n + 1) (2n + 1)$

$S = n(n + 1)(2n + 1) / 6$

Thus sum of the squares of the 1^{st}, n natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$

i.e. $1^2 + 2^2 + 3^2 + \dots\dots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

Similarly, sum of the cubes of 1^{st} n natural number can be found out as $\left\{\dfrac{n(n+1)}{2}\right\}^2$ by taking the identity

$m^4 - (m-1)^4 = 4m^3 - 6m^2 + 4m - 1$ and putting m = 1, 2, 3,…., n.

Thus

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

Exercise 6 (A)

Choose the most appropriate option (a), (b) , (c) or (d)

1. The nth element of the sequence 1, 3, 5, 7,……..Is
(a) n (b) 2n – 1 (c) 2n +1 (d) none of these

2. The nth element of the sequence –1, 2, –4, 8 ….. is
(a) $(-1)^n 2^{n-1}$ (b) 2^{n-1} (c) 2^n (d) none of these

3. $\sum_{i=4}^{7}\sqrt{2i-1}$ can be written as
(a) $\sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$ (b) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$
(c) $2\sqrt{7}+2\sqrt{9}+2\sqrt{11}+2\sqrt{13}$ (d) none of these.

4. –5, 25, –125 , 625, ….. can be written as
(a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}5^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) none of these

5. The first three terms of sequence when nth term t_n is $n^2 - 2n$ are
(a) –1, 0, 3 (b) 1, 0, 2 (c) –1, 0, –3 (d) none of these

6. Which term of the progression –1, –3, –5, …. Is –39
(a) 21^{st} (b) 20^{th} (c) 19^{th} (d) none of these

7. The value of x such that 8x + 4, 6x – 2, 2x + 7 will form an AP is
(a) 15 (b) 2 (c) 15/2 (d) none of the these

8. The mth term of an A. P. is n and nth term is m. The r th term of it is
(a) m + n +r (b) n + m – 2r (c) m + n + r/2 (d) m + n – r

9. The number of the terms of the series $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + \ldots\ldots\ldots\ldots$will amount to 155 is
(a) 30 (b) 31 (c) 32 (d) none of these

10. The nth term of the series whose sum to n terms is $5n^2 + 2n$ is
(a) 3n – 10 (b) 10n – 2 (c) 10n – 3 (d) none of these

11. The 20^{th} term of the progression 1, 4, 7, 10.................is
(a) 58 (b) 52 (c) 50 (d) none of these

12. The last term of the series 5, 7, 9,….. to 21 terms is
(a) 44 (b) 43 (c) 45 (d) none of these

13. The last term of the A.P. 0.6, 1.2, 1.8,… to 13 terms is
(a) 8.7 (b) 7.8 (c) 7.7 (d) none of these
14. The sum of the series 9, 5, 1,…. to 100 terms is
(a) –18900 (b) 18900 (c) 19900 (d) none of these
15. The two arithmetic means between –6 and 14 is
(a) 2/3, 1/3 (b) 2/3, $7\frac{1}{3}$ (c) – 2/3, $-7\frac{1}{3}$ (d) none of these
16. The sum of three integers in AP is 15 and their product is 80. The integers are
(a) 2, 8, 5 (b) 8, 2, 5 (c) 2, 5, 8 (d) 8, 5, 2
17. The sum of n terms of an AP is $3n^2 + 5n$. The series is
(a) 8, 14, 20, 26 (b) 8, 22, 42, 68 (c) 22, 68, 114, ... (d) none of these
18. The number of numbers between 74 and 25556 divisible by 5 is
(a) 5090 (b) 5097 (c) 5095 (d) none of these
19. The pth term of an AP is (3p – 1)/6. The sum of the first n terms of the AP is
(a) n (3n + 1) (b) n/12 (3n + 1) (c) n/12 (3n – 1) (d) none of these
20. The arithmetic mean between 33 and 77 is
(a) 50 (b) 45 (c) 55 (d) none of these
21. The 4 arithmetic means between –2 and 23 are
(a) 3, 13, 8, 18 (b) 18, 3, 8, 13 (c) 3, 8, 13, 18 (d) none of these
22. The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal is magnitude but opposite in sign. The 3rd term of the AP is
(a) $6\frac{4}{11}$ (b) 6 (c) 4/11 (d) none of these
23. The sum of a certain number of terms of an AP series –8, –6, –4, …… is 52. The number of terms is
(a) 12 (b) 13 (c) 11 (d) none of these
24. The 1st and the last term of an AP are –4 and 146. The sum of the terms is 7171. The number of terms is
(a) 101 (b) 100 (c) 99 (d) none of these
25. The sum of the series 3 ½ + 7 + 10 ½ + 14 + …. To 17 terms is
(a) 530 (b) 535 (c) 535 ½ (d) none of these

6.4 GEOMETRIC PROGRESSION (G.P.)

If in a sequence of terms each term is constant multiple of the proceeding term, then the sequence is called a Geometric Progression (G.P). The constant multiplier is called the *common ratio*

Examples: 1) In 5, 15, 45, 135,….. common ratio is 15/5 = 3

2) In 1, $^1/_2$, $^1/_4$, $^1/_8$, … common ratio is (½) /1=½

3) In 2, –6, 18, –54, …. common ratio is (–6) / 2 =–3

Illustrations: Consider the following series :–

(i) 1 + 4 + 16 + 64 + …………….

Here second term / 1st term = 4/1 = 4; third term / second term = 16/4 = 4

fourth term/third term = 64/16 = 4 and so on.

Thus, we find that, in the entire series, the ratio of any term and the term preceding it, is a constant.

(ii) 1/3 – 1/9 + 1/27 – 1/81 + ………….

Here second term / 1st term = (–1/9) / (1/3) = –1/3

third term / second term = (1/27) / (–1/9) = –1/3

fourth term / third term = (–1/81) / (1/27) = –1/3 and so on.

Here also, in the entire series, the ratio of any term and the term preceding one is constant.

The above mentioned series are known as **Geometric Series.**

Let us consider the sequence a, ar, ar^2, ar^3, ….

1st term = a, 2nd term = ar = ar^{2-1}, 3rd term = $ar^2 = ar^{3-1}$, 4th term = $ar^3 = ar^{4-1}$, …..

Similarly nth term of GP $t_n = ar^{n-1}$

Thus, common ratio = $\dfrac{\text{Any term}}{\text{Preceding term}} = \dfrac{t_n}{t_{n-1}}$

$= ar^{n-1}/ar^{n-2} = r$

Thus, general term of a G.P is given by ar^{n-1} and the general form of G.P. is

$a + ar + ar^2 + ar^3 + \ldots\ldots \ldots$

For example, $r = \dfrac{t_2}{t_1} = \dfrac{ar}{a}$

So $r = \dfrac{t_2}{t_1} = \dfrac{t_3}{t_2} = \dfrac{t_4}{t_3} = \ldots.$

Example 1: If a, ar, ar^2, ar^3, …. be in G.P. Find the common ratio.

Solution: 1st term = a, 2nd term = ar

Ratio of any term to its preceding term = ar/a = r = common ratio.

Example 2: Which term of the progression 1, 2, 4, 8,… is 256?

Solution : a = 1, r = 2/1 = 2, n = ? $t_n = 256$

$t_n = ar^{n-1}$

or $256 = 1 \times 2^{n-1}$ i.e., $2^8 = 2^{n-1}$ or, n – 1 = 8 i.e., n = 9

Thus 9^{th} term of the G. P. is 256

6.5 GEOMETRIC MEAN

If a, b, c are in G.P we get $b/a = c/b \Rightarrow b^2 = ac$, b is called the geometric mean between a and c

Example 1: Insert 3 geometric means between 1/9 and 9.

Solution: 1/9, –, –, –, 9

$a = 1/9, r = ?, n = 2 + 3 = 5, t_n = 9$

we know $t_n = ar^{n-1}$

or $1/9 \times r^{5-1} = 9$

or $r^4 = 81 = 3^4 \Rightarrow r = 3$

Thus 1^{st} G. M = $1/9 \times 3 = 1/3$

2^{nd} G. M = $1/3 \times 3 = 1$

3^{rd} G. M = $1 \times 3 = 3$

Example 2: Find the G.P where 4^{th} term is 8 and 8^{th} term is 128/625

Solution : Let a be the 1^{st} term and r be the common ratio.

By the question $t_4 = 8$ and $t_8 = 128/625$

So $ar^3 = 8$ and $ar^7 = 128 / 625$

Therefore $ar^7 / ar^3 = \dfrac{128}{625 \times 8} \Rightarrow r^4 = 16 / 625 = (\pm 2/5)^4 \Rightarrow r = 2/5$ and $-2/5$

Now $ar^3 = 8 \Rightarrow a \times (2/5)^3 = 8 \Rightarrow a = 125$

Thus the G. P is

125, 50, 20, 8, 16/5, ………..

When r = –2/5 , a = –125 and the G.P is –125, 50, –20, 8, –16/5 ,………

Finally, the G.P. is 125, 50, 20, 8, 16/5, ………..

or, –125, 50, –20, 8, –16/5,………

Sum of first n terms of a G P

Let a be the 1^{st} term and r be the common ratio. So the 1^{st} n terms are a, ar, ar^2, …... ar^{n-1}.

If S be the sum of n terms,

$S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$ .... (i)

Now $rS_n = ar + ar^2 + \ldots\ldots + ar^{n-1} + ar^n$ .... (ii)

Subtracting (i) from (ii)

$S_n - rS_n = a - ar^n$

or $S_n(1 - r) = a(1 - r^n)$

or $S_n = a(1 - r^n) / (1 - r)$ when $r < 1$

$S_n = a(r^n - 1) / (r - 1)$ when $r > 1$

If r = 1 , then $S_n = a + a + a + \ldots\ldots$ to n terms

$= na$

If the nth term of the G. P be l then $\ell = ar^{n-1}$

Therefore, $S_n = (ar^n - a) / (r - 1) = (a r^{n-1} r - a) / (r - 1) = \dfrac{lr - a}{r - 1}$

So, when the last term of the G. P is known, we use this formula.

Sum of infinite geometric series

$S = a(1 - r^n) / (1 - r)$ when $r < 1$

$= a(1 - 1/R^n) / (1 - 1/R)$ (since $r < 1$, we take $r = 1/R$).

If $n \to \infty, 1/R^n \to 0$

Thus $S_\mu = \dfrac{a}{1-r},\ r < 1$

i.e. Sum of G.P. upto infinity is $\dfrac{a}{1-r}$, where $r < 1$

Also, $S_\mu = \dfrac{a}{1-r}$, if $-1<r<1$.

Example 1: Find the sum of 1 + 2 + 4 + 8 + … to 8 terms.,

Solution: Here $a = 1, r = 2/1 = 2, n = 8$

Let $S = 1 + 2 + 4 + 8 + \ldots\ldots$ to 8 terms

$= 1(2^8 - 1) / (2 - 1) = 2^8 - 1 = 255$

Example 2: Find the sum to n terms of $6 + 27 + 128 + 629 + \ldots\ldots$

Solution: Required Sum $= (5 + 1) +)(5^2 + 2) + (5^3 + 3)(5^4 + 4) + \ldots$ to n terms

$= (5 + 5^2 + 5^3 + \ldots\ldots + 5^n) + (1 + 2 + 3 + .. + n \text{ terms})$

$= \{5(5^n - 1) / (5 - 1)\} + \{n(n + 1) / 2\}$

$= \{5(5^n - 1) / 4\} + \{n(n + 1) / 2\}$

Example 3: Find the sum to n terms of the series

$3 + 33 + 333 + \ldots\ldots$

Solution: Let S denote the required sum.

i.e. $S = 3 + 33 + 333 + \ldots\ldots\ldots\ldots..$ to n terms

$= 3\,(1 + 11 + 111 + \ldots\ldots.$ to n terms)

$= \frac{3}{9}\,(9 + 99 + 999 + \ldots.$ to n terms)

$= \frac{3}{9}\,\{(10-1) + (10^2-1) + (10^3-1) + \ldots + (10^n-1)\}$

$= \frac{3}{9}\,\{(10 + 10^2 + 10^3 + \ldots. + 10^n) - n\}$

$= \frac{3}{9}\,\{10\,(1 + 10 + 10^2 + \ldots + 10^{n-1}) - n\}$

$= \frac{3}{9}\,[\{10\,(10^n-1)\,/\,(10-1)\} - n]$

$= \frac{3}{81}\,(10^{n+1} - 10 - 9n)$

$= \frac{1}{27}\,(10^{n+1} - 9n - 10)$

Example 4: Find the sum of n terms of the series 0.7 + 0.77 + 0.777 + …. to n terms

Solution : Let S denote the required sum.

i.e. $S = 0.7 + 0.77 + 0.777 + \ldots..$ to n terms

$= 7\,(0.1 + 0.11 + 0.111 + \ldots.$ to n terms)

$= \frac{7}{9}\,(0.9 + 0.99 + 0.999 + \ldots$ to n terms)

$= \frac{7}{9}\,\{(1 - 1/10) + (1 - 1/10^2) + (1 - 1/10^3) + \ldots + (1 - 1/10^n)\}$

$= \frac{7}{9}\,\{n - \frac{1}{10}\,(1 + 1/10 + 1/10^2 + \ldots. + 1/10^{n-1})\}$

So $S = \frac{7}{9}\{n - \frac{1}{10}\,(1 - 1/10^n)/(1 - 1/10)\}$

$= \frac{7}{9}\{n - (1 - 10^{-n})\,/\,9)\}$

$= \frac{7}{81}\,\{9n - 1 + 10^{-n}\}$

Example 5: Evaluate $0.21\dot{7}\dot{5}$ using the sum of an infinite geometric series.

Solution: $0.21\dot{7}\dot{5}$ = 0.2175757575 …….

$0.21\dot{7}\dot{5}$ = 0.21 + 0.0075 + 0.000075 + ….

= 0.21 + 75 (1 + 1/10^2 + 1/10^4 + ….) / 10^4

= 0.21 + 75 {1 / (1– 1/10^2} / 10^4

= 0.21 + (75/10^4) × 10^2 /99

=21/100 + (¾) × (1/99)

= 21/100 + 1/132

= (693 + 25)/3300 = 718/3300 = 359/1650

Example 6: Find three numbers in G. P whose sum is 19 and product is 216.

Solution: Let the 3 numbers be a/r, a, ar.

According to the question a/r × a × ar = 216

or $a^3 = 6^3$ => a =6

So the numbers are 6/r, 6, 6r

Again 6/r + 6 + 6r = 19

or 6/r + 6r = 13

or 6 + 6r^2 = 13r

or 6r^2 – 13r + 6 = 0

or 6r^2 – 4r – 9r + 6 = 0

or 2r(3r –2) – 3 (3r – 2) = 2

or (3r – 2) (2r – 3) = 0 or, r = 2/3 , 3/2

So the numbers are

6/(2/3), 6, 6 × (2/3) = 9 , 6 , 4

or 6/(3/2), 6 , 6 × (3/2) = 4 , 6 , 9

Exercise 6 (B)

Choose the most appropriate option (a), (b), (c) or (d)

1. The 7th term of the series 6, 12, 24,……is
   (a) 384 (b) 834 (c) 438 (d) none of these
2. t_8 of the series 6, 12, 24,…is
   (a) 786 (b) 768 (c) 867 (c) none of these
3. t_{12} of the series –128, 64, –32, ….is
   (a) – 1/16 (b) 16 (c) 1/16 (d) none of these
4. The 4th term of the series 0.04, 0.2, 1, … is
   (a) 0.5 (b) ½ (c) 5 (d) none of these

5. The last term of the series 1, 2, 4,…. to 10 terms is
(a) 512 (b) 256 (c) 1024 (d) none of these

6. The last term of the series 1, –3, 9, –27 up to 7 terms is
(a) 297 (b) 729 (c) 927 (d) none of these

7. The last term of the series x^2, x, 1, …. to 31 terms is
(a) x^{28} (b) $1/x$ (c) $1/x^{28}$ (d) none of these

8. The sum of the series –2, 6, –18, …. to 7 terms is
(a) –1094 (b) 1094 (c) – 1049 (d) none of these

9. The sum of the series 24, 3, 8, 1, 2, 7,… to 8 terms is
(a) 36 (b) $\left(36\frac{13}{30}\right)$ (c) $36\frac{1}{9}$ (d) none of these

10. The sum of the series $\frac{1}{\sqrt{3}}+1+\frac{3}{\sqrt{3}}+$ ……to 18 terms is
(a) $9841\,\frac{(1+\sqrt{3})}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) none of these

11. The second term of a G P is 24 and the fifth term is 81. The series is
(a) 16, 36, 24, 54,.. (b) 24, 36, 53,… (c) 16, 24, 36, 54,.. (d) none of these

12. The sum of 3 numbers of a G P is 39 and their product is 729. The numbers are
(a) 3, 27, 9 (b) 9, 3, 27 (c) 3, 9, 27 (d) none of these

13. In a G. P, the product of the first three terms 27/8. The middle term is
(a) 3/2 (b) 2/3 (c) 2/5 (d) none of these

14. If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be
(a) Rs. 163 (b) Rs. 183 (c) Rs. 163.83 (d) none of these

15. Sum of n terms of the series 4 + 44 + 444 + … is
(a) $4/9 \{ 10/9 (10^n -1) -n \}$ (b) $10/9 (10^n -1) -n$
(c) $4/9 (10^n -1) -n$ (d) none of these

16. Sum of n terms of the series 0.1 + 0.11 + 0.111 + … is
(a) $1/9 \{n - (1- (0.1)^n)\}$ (b) $1/9 \{n - (1-(0.1)^n)/9\}$
(c) $n- 1 - (0.1)^n/9$ (d) none of these

17. The sum of the first 20 terms of a G. P is 244 times the sum of its first 10 terms. The common ratio is
(a) $\pm\sqrt{3}$ (b) ±3 (c) $\sqrt{3}$ (d) none of these

18. Sum of the series 1 + 3 + 9 + 27 +….is 364. The number of terms is
(a) 5 (b) 6 (c) 11 (d) none of these

19. The product of 3 numbers in G P is 729 and the sum of squares is 819. The numbers are
    (a) 9, 3, 27 (b) 27, 3, 9 (c) 3, 9, 27 (d) none of these

20. The sum of the series 1 + 2 + 4 + 8 + .. to n term
    (a) $2^n - 1$ (b) $2n - 1$ (c) $1/2^n - 1$ (d) none of these

21. The sum of the infinite GP 14, – 2, + 2/7, – 2/49, + … is
    (a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) none of these

22. The sum of the infinite G. P. 1 - 1/3 + 1/9 - 1/27 +... is
    (a) 0.33 (b) 0.57 (c) 0.75 (d) none of these

23. The number of terms to be taken so that 1 + 2 + 4 + 8 + will be 8191 is
    (a) 10 (b) 13 (c) 12 (d) none of these

24. Four geometric means between 4 and 972 are
    (a) 12,30,100,324 (b) 12,24,108,320 (c) 10,36,108,320 (d) none of these

Illustrations :

(I) A person is employed in a company at Rs. 3000 per month and he would get an increase of Rs. 100 per year. Find the total amount which he receives in 25 years and the monthly salary in the last year.

Solution:

He gets in the 1st year at the Rate of 3000 per month;

In the 2nd year he gets at the rate of Rs. 3100 per month;

In the 3rd year at the rate of Rs. 3200 per month so on.

In the last year the monthly salary will be

Rs. {3000 + (25 – 1) × 100} = Rs. 5400

Total amount = Rs. 12 (3000 + 3100 + 3200 +… + 5400) $\left[\text{Use } S_n = \frac{n}{2}(a + l)\right]$

= Rs. 12 × 25/2 (3000 + 5400)

= Rs. 150 × 8400

= Rs. 12,60,000

(II) A person borrows Rs. 8,000 at 2.76% Simple Interest per annum. The principal and the interest are to be paid in the 10 monthly instalments. If each instalment is double the preceding one, find the value of the first and the last instalment.

Solution:

Interest to be paid = 2.76 × 10 × 8000 / 100 × 12 = Rs. 184

Total amount to be paid in 10 monthly instalment is Rs. (8000 + 184) = Rs. 8184

The instalments form a G P with common ratio 2 and so Rs. 8184 = a $(2^{10} - 1) / (2 - 1)$,

a = 1st instalment

Here a = Rs. 8184 / 1023 = Rs. 8

The last instalment = $ar^{10-1} = 8 \times 2^9 = 8 \times 512$ = Rs. 4096

Exercise 6 (c)

Choose the most appropriate option (a), (b), (c) or (d)

1. Three numbers are in AP and their sum is 21. If 1, 5, 15 are added to them respectively, they form a G. P. The numbers are
(a) 5, 7, 9 (b) 9, 5, 7 (c) 7, 5, 9 (d) none of these

2. The sum of $1 + 1/3 + 1/3^2 + 1/3^3 + \ldots + 1/3^{n-1}$ is
(a) 2/3 (b) 3/2 (c) 4/5 (d) none of these

3. The sum of the infinite series 1 + 2/3 + 4/9 + .. is
(a) 1/3 (b) 3 (c) 2/3 (d) none of these

4. The sum of the first two terms of a G.P. is 5/3 and the sum to infinity of the series is 3. The common ratio is
(a) 1/3 (b) 2/3 (c) – 2/3 (d) none of these

5. If p, q and r are in A.P. and x, y, z are in G.P. then $x^{q-r}. y^{r-p}. z^{p-q}$ is equal to
(a) 0 (b) –1 (c) 1 (d) none of these

6. The sum of three numbers in G.P. is 70. If the two extremes by multiplied each by 4 and the mean by 5, the products are in AP. The numbers are
(a) 12, 18, 40 (b) 10, 20, 40 (c) 40, 20, 10 (d) none of these

7. The sum of 3 numbers in A.P. is 15. If 1, 4 and 19 be added to them respectively, the results are is G. P. The numbers are
(a) 26, 5, –16 (b) 2, 5, 8 (c) 5, 8, 2 (d) none of these

8. Given x, y, z are in G.P. and $x^p = y^q = z^\sigma$, then $1/p$, $1/q$, $1/\sigma$ are in
(a) A.P. (b) G.P. (c) Both A.P. and G.P. (d) none of these

9. If the terms 2x, (x+10) and (3x+2) be in A.P., the value of x is
(a) 7 (b) 10 (c) 6 (d) none of these

10. If A be the A.M. of two positive unequal quantities x and y and G be their G. M, then
(a) $A < G$ (b) $A > G$ (c) $A \geq G$ (d) $A \leq G$

11. The A.M. of two positive numbers is 40 and their G. M. is 24. The numbers are
(a) (72, 8) (b) (70, 10) (c) (60, 20) (d) none of these

12. Three numbers are in A.P. and their sum is 15. If 8, 6, 4 be added to them respectively, the numbers are in G.P. The numbers are
(a) 2, 6, 7 (b) 4, 6, 5 (c) 3, 5, 7 (d) none of these

13. The sum of four numbers in G. P. is 60 and the A.M. of the 1st and the last is 18. The numbers are
(a) 4, 8, 16, 32 (b) 4, 16, 8, 32 (c) 16, 8, 4, 20 (d) none of these

14. A sum of Rs. 6240 is paid off in 30 instalments such that each instalment is Rs. 10 more than the proceeding installment. The value of the 1st instalment is
(a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) none of these

15. The sum of 1.03 + $(1.03)^2$ + $(1.03)^3$ + …. to n terms is
(a) 103 $\{(1.03)^n - 1\}$ (b) 103/3 $\{(1.03)^n - 1\}$ (c) $(1.03)^n - 1$ (d) none of these

16. If x, y, z are in A.P. and x, y, (z + 1) are in G.P. then
(a) $(x - z)^2 = 4x$ (b) $z^2 = (x - y)$ (c) $z = x - y$ (d) none of these

17. The numbers x, 8, y are in G.P. and the numbers x, y, –8 are in A.P. The value of x and y are
(a) (–8, –8) (b) (16, 4) (c) (8, 8) (d) none of these

18. The nth term of the series 16, 8, 4,…. Is $1/2^{17}$. The value of n is
(a) 20 (b) 21 (c) 22 (d) none of these

19. The sum of n terms of a G.P. whose first terms 1 and the common ratio is $^1/_2$, is equal to $1\frac{127}{128}$. The value of n is
(a) 7 (b) 8 (c) 6 (d) none of these

20. t_4 of a G.P. in x, t_{10} = y and t_{16} = z. Then
(a) $x^2 = yz$ (b) $z^2 = xy$ (c) $y^2 = zx$ (d) none of these

21. If x, y, z are in G.P., then
(a) $y^2 = xz$ (b) $y (z^2 + x^2) = x (z^2 + y^2)$ (c) $2y = x+z$ (d) none of these

22. The sum of all odd numbers between 200 and 300 is
(a) 11600 (b) 12490 (c) 12500 (d) 24750

23. The sum of all natural numbers between 500 and 1000 which are divisible by 13, is
(a) 28405 (b) 24805 (c) 28540 (d) none of these

24. If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9,…... the resulting sum is
(a) ‘a’ perfect cube (b) ‘a’ perfect square (c) ‘a’ number (d) none of these

25. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is
(a) 10200 (b) 15200 (c) 16200 (d) none of these

26. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 and 5 is
(a) 2200 (b) 2000 (c) 2220 (d) none of these

27. A person pays Rs. 975 by monthly instalment each less then the former by Rs. 5. The first instalment is Rs. 100. The time by which the entire amount will be paid is
(a) 10 months (b) 15 months (c) 14 months (d) none of these

28. A person saved Rs. 16,500 in ten years. In each year after the first year he saved Rs. 100 more than he did in the preceding year. The amount of money he saved in the 1^{st} year was

(a) Rs. 1000 (b) Rs. 1500 (c) Rs. 1200 (d) none of these

29. At 10% C.I. p.a., a sum of money accumulate to Rs. 9625 in 5 years. The sum invested initially is

(a) Rs. 5976.37 (b) Rs. 5970 (c) Rs. 5975 (d) Rs. 5370.96

30. The population of a country was 55 crose in 2005 and is growing at 2% p.a C.I. the population is the year 2015 is estimated as

(a) 5705 (b) 6005 (c) 6700 (d) none of these

ANSWERS

Exercise 6 (A)

1. b	2. a	3. a	4. a	5. a	6. b	7. c	8. d
9. a, b	10 c	11. a	12. c	13. b	14. a	15. b	16. c, d
17. a	18. b	19. b	20. c	21. c	22. a	23. b	24. a
25. c							

Exercise 6 (B)

1. a	2. b	3. c	4. c	5. a	6. b	7. c	8. a
9. b	10. a	11. c	12. c	13. a	14. c	15. a	16. b
17. a	18. b	19. c	20. a	21. b	22. c	23. b	24. d

Exercise 6 (C)

1. a	2. d	3. b	4. b, c	5. c	6. b, c	7. a, b	8. a
9. c	10. b	11. a	12. c	13. a	14. d	15. b	16. a
17. b	18. c	19. b	20. c	21. a	22. d	23. a	24. b
25. c	26. a	27. b	28. c	29. d	30. d		

ADDITIONAL QUESTION BANK

1. If a, b, c are in A.P. as well as in G.P. then –

   (A) They are also in H.P. (Harmonic Progression) (B) Their reciprocals are in A.P.

   (C) Both (A) and (B) are true (D) Both (A) and (B) are false

2. If a, b, c be respectively $p^{th,}$ q^{th} and r^{th} terms of an A.P. the value of $a(q-r)+b(r-p)+c(p-q)$ is _________.

   (A) 0 (B) 1 (C) –1 (D) None

3. If the p^{th} term of an A.P. is q and the q^{th} term is p the value of the r^{th} term is_________.

   (A) $p - q - r$ (B) $p + q - r$

   (C) $p + q + r$ (D) None

4. If the p^{th} term of an A.P. is q and the q^{th} term is p the value of the $(p + q)^{th}$ term is_______.

   (A) 0 (B) 1 (C) –1 (D) None

5. The sum of first n natural number is _______.

   (A) $(n/2)(n+1)$ (B) $(n/6)(n+1)(2n+1)$
   (C) $[(n/2)(n+1)]^2$ (D) None

6. The sum of square of first n natural number is __________.

   (A) $(n/2)(n+1)$ (B) $(n/6)(n+1)(2n+1)$
   (C) $[(n/2)(n+1)]^2$ (D) None

7. The sum of cubes of first n natural number is __________.

   (A) $(n/2)(n+1)$ (B) $(n/6)(n+1)(2n+1)$
   (C) $[(n/2)(n+1)]^2$ (D) None

8. The sum of a series in A.P. is 72 the first term being 17 and the common difference –2. the number of terms is __________.

   (A) 6 (B) 12 (C) 6 or 12 (D) None

9. Find the sum to n terms of $(1-1/n) + (1-2/n) + (1-3/n) + ...$

   (A) $\frac{1}{2}(n-1)$ (B) $\frac{1}{2}(n+1)$ (C) $(n-1)$ (D) $(n+1)$

10. If Sn the sum of first n terms in a series is given by $2n^2 + 3n$ the series is in ______.

    (A) A.P. (B) G.P. (C) H.P. (D) None

11. The sum of all natural numbers between 200 and 400 which are divisible by 7 is ______.

(A) 7730 (B) 8729 (C) 7729 (D) 8730

12. The sum of natural numbers upto 200 excluding those divisible by 5 is ________.

(A) 20100 (B) 4100 (C) 16000 (D) None

13. If a, b, c be the sums of $p\ q\ r$ terms respectively of an A.P. the value of $(a/p)\,(q-r)+(b/q)\,(r-p)+(c/r)\,(p-q)$ is ______.

(A) 0 (B) 1 (C) –1 (D) None

14. If S_1, S_2, S_3 be the respectively the sum of terms of $n, 2n, 3n$ an A.P. the value of $S_3 \div (S_2 - S_1)$ is given by ______.

(A) 1 (B) 2 (C) 3 (D) None

15. The sum of n terms of two A.P.s are in the ratio of $(7n-5)/(5n+17)$. Then the _______ term of the two series are equal.

(A) 12 (B) 6 (C) 3 (D) None

16. Find three numbers in A.P. whose sum is 6 and the product is –24

(A) –2, 2, 6 (B) –1, 1, 3 (C) 1, 3, 5 (D) 1, 4, 7

17. Find three numbers in A.P. whose sum is 6 and the sum of whose square is 44.

(A) –2, 2, 6 (B) –1, 1, 3 (C) 1, 3, 5 (D) 1, 4, 7

18. Find three numbers in A.P. whose sum is 6 and the sum of their cubes is 232.

(A) –2, 2, 6 (B) –1, 1, 3 (C) 1, 3, 5 (D) 1, 4, 7

19. Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio of 2:3

(A) 2, 2.25, 2.5, 2.75, 3 (B) –2, –2.25, –2.5, –2.75, –3

(C) 4, 4.5, 5, 5.5, 6 (D) –4, –4.5, –5, –5.5, –6

20. If a, b, c are in A.P. then the value of $(a^3+4b^3+c^3)/[b(a^2+c^2)]$ is

(A) 1 (B) 2 (C) 3 (D) None

21. If a, b, c are in A.P. then the value of $(a^2+4ac+c^2)/(ab+bc+ca)$ is

(A) 1 (B) 2 (C) 3 (D) None

22. If *a, b, c* are in A.P. then $(a/bc)(b+c), (b/ca)(c+a), (c/ab)(a+b)$ are in ____________.

(A) A.P. (B) G.P. (C) H.P. (D) None

23. If *a, b, c* are in A.P. then $a^2(b+c), b^2(c+a), c^2(a+b)$ are in ________.

(A) A.P. (B) G.P. (C) H.P. (D) None

24. If $(b+c)^{-1}, (c+a)^{-1}, (a+b)^{-1}$ are in A.P. then a^2, b^2, c^2 are in _________.

(A) A.P. (B) G.P. (C) H.P. (D) None

25. If a^2, b^2, c^2 are in A.P. then $(b+c), (c+a), (a+b)$ are in ________.

(A) A.P. (B) G.P. (C) H.P. (D) None

26. If a^2, b^2, c^2 are in A.P. then $a/(b+c), b/(c+a), c/(a+b)$ are in ____________.

(A) A.P. (B) G.P. (C) H.P. (D) None

27. If $(b+c-a)/a, (c+a-b)/b, (a+b-c)/c$ are in A.P. then a, b, c are in __________.

(A) A.P. (B) G.P. (C) H.P. (D) None

28. If $(b-c)^2, (c-a)^2, (a-b)^2$ are in A.P. then $(b-c), (c-a), (a-b)$ are in _______.

(A) A.P. (B) G.P. (C) H.P. (D) None

29. If *a b c* are in A.P. then $(b+c), (c+a), (a+b)$ are in ________.

(A) A.P. (B) G.P. (C) H.P. (D) None

30. Find the number which should be added to the sum of any number of terms of the A.P. 3, 5, 7, 9, 11 …….resulting in a perfect square.

(A) –1 (B) 0 (C) 1 (D) None

31. The sum of *n* terms of an A.P. is $2n^2 + 3n$. Find the n^{th} term.

(A) 4n + 1 (B) 4n - 1 (C) 2n + 1 (D) 2n - 1

32. The p^{th} term of an A.P. is $1/q$ and the q^{th} term is $1/p$. The sum of the pq^{th} term is_______.

(A) $\frac{1}{2}(pq+1)$ (B) $\frac{1}{2}(pq-1)$ (C) pq+1 (D) pq-1

33. The sum of p terms of an A.P. is q and the sum of q terms is p. The sum of $p + q$ terms is ________.

(A) $-(p + q)$ (B) $p + q$ (C) $(p - q)^2$ (D) $P^2 - q^2$

34. If S_1, S_2, S_3 be the sums of n terms of three A.P.s the first term of each being unity and the respective common differences 1, 2, 3 then $(S_1 + S_3) / S_2$ is ______.

(A) 1 (B) 2 (C) –1 (D) None

35. The sum of all natural numbers between 500 and 1000, which are divisible by 13, is _______.

(A) 28400 (B) 28405 (C) 28410 (D) None

36. The sum of all natural numbers between 100 and 300, which are divisible by 4, is ____.

(A) 10200 (B) 30000 (C) 8200 (D) 2200

37. The sum of all natural numbers from 100 to 300 excluding those, which are divisible by 4, is _______.

(A) 10200 (B) 30000 (C) 8200 (D) 2200

38. The sum of all natural numbers from 100 to 300, which are divisible by 5, is ______.

(A) 10200 (B) 30000 (C) 8200 (D) 2200

39. The sum of all natural numbers from 100 to 300, which are divisible by 4 and 5, is ______.

(A) 10200 (B) 30000 (C) 8200 (D) 2200

40. The sum of all natural numbers from 100 to 300, which are divisible by 4 or 5, is ______.

(A) 10200 (B) 8200 (C) 2200 (D) 16200

41. If the n terms of two A.P.s are in the ratio (3n+4) : (n+4) the ratio of the fourth term is ______.

(A) 2 (B) 3 (C) 4 (D) None

42. If a, b, c, d are in A.P. then

(A) $a^2 - 3b^2 + 3c^2 - d^2 = 0$ (B) $a^2 + 3b^2 + 3c^2 + d^2 = 0$ (C) $a^2 + 3b^2 + 3c^2 - d^2 = 0$ (D) None

43. If a, b, c, d, e are in A.P. then

(A) $a - b - d + e = 0$ (B) $a - 2c + e = 0$ (C) $b - 2c + d = 0$ (D) all the above

44. The three numbers in A.P. whose sum is 18 and product is 192 are _______.

(A) 4, 6, 8 (B) –4, –6, –8 (C) 8, 6, 4

(D) both (A) and (C)

45. The three numbers in A.P., whose sum is 27 and the sum of their squares is 341, are _______.

(A) 2, 9, 16 (B) 16, 9, 2 (C) both (A) and (B) (D) –2, –9, –16

46. The four numbers in A.P., whose sum is 24 and their product is 945, are _______.

(A) 3, 5, 7, 9 (B) 2, 4, 6, 8 (C) 5, 9, 13, 17 (D) None

47. The four numbers in A.P., whose sum is 20 and the sum of their squares is 120, are _______.

(A) 3, 5, 7, 9 (B) 2, 4, 6, 8 (C) 5, 9, 13, 17 (D) None

48. The four numbers in A.P. with the sum of second and third being 22 and the product of the first and fourth beinf 85 are _______.

(A) 3, 5, 7, 9 (B) 2, 4, 6, 8 (C) 5, 9, 13, 17 (D) None

49. The five numbers in A.P. with their sum 25 and the sum of their squares 135 are _______.

(A) 3, 4, 5, 6, 7 (B) 3, 3.5, 4, 4.5, 5 (C) –3, –4, –5, –6, –7

(D) –3, –3.5, –4, –4.5, –5

50. The five numbers in A.P. with the sum 20 and product of the first and last 15 are _______.

(A) 3, 4, 5, 6, 7 (B) 3, 3.5, 4, 4.5, 5 (C) –3, –4, –5, –6, –7

(D) –3, –3.5, –4, –4.5, –5

51. The sum of n terms of 2, 4, 6, 8….. is

(A) n(n+1) (B) (n/2)(n+1) (C) n(n–1) (D) (n/2)(n-1)

52. The sum of n terms of a+b, 2a, 3a–b, ….. is

(A) n(a–b)+2b (B) n(a+b) (C) both the above (D) None

53. The sum of n terms of $(x + y)^2$, $(x^2 + y^2)$, $(x - y)^2$,...... is
(A) $(x + y)^2 - 2(n - 1)xy$ (B) $n(x + y)^2 - n(n - 1)xy$ (C) both the above (D) None

54. The sum of n terms of (1/n)(n–1), (1/n) (n–2), (1/n) (n–3)....... is
(A) 0 (B) (1/2)(n–1) (C) (1/2)(n+1) (D) None

55. The sum of n terms of 1.4, 3.7, 5.10 ……. Is
(A) $(n/2)(4n^2+5n-1)$ (B) $n(4n^2+5n-1)$ (C) $(n/2)(4n^2-5n-1)$ (D) None

56. The sum of n terms of $1^2, 3^2, 5^2, 7^2,.......$is

(A) $(n/3)(4n^2 -1)$ (B) $(n/2)(4n^2 -1)$ (C) $(n/3)(4n^2 +1)$ (D) None

57. The sum of n terms of 1, (1 + 2), (1 + 2 + 3) …….. is

(A) $(n/3)(n+1)(n-2)$ (B) $(n/3)(n+1)(n+2)$ (C) $n(n+1)(n+2)$ (D) None

58. The sum of n terms of the series $1^2/1+(1^2+2^2)/2+(1^2+2^2+3^2)/3+.......$is

(A) $(n/36)(4n^2 +15n+17)$ (B) $(n/12)(4n^2+15n+17)$
(C) $(n/12)(4n^2 +15n+17)$ (D) None

59. The sum of n terms of the series 2.4.6 + 4.6.8 + 6.8.10 + ………. is

(A) $2n(n^3+6n^2+11n+6)$ (B) $2n(n^3-6n^2+11n-6)$
(C) $n(n^3+6n^2+11n+6)$ (D) $n(n^3+6n^2+11n-6)$

60. The sum of n terms of the series $1.3^2+4.4^2+7.5^2+10.6^2+........$ is

(A) $(n/12)(n+1)(9n^2+49n+44)-8n$ (B) $(n/12)(n+1)(9n^2+49n+44)+8n$
(C) $(n/6)(2n+1)(9n^2+49n+44)-8n$ (D) None

61. The sum of n terms of the series 4 + 6 + 9 + 13 …….. is

(A) $(n/6)(n^2+3n+20)$ (B) $(n/6)(n+1)(n+2)$ (C) $(n/3)(n+1)(n+2)$ (D) None

62. The sum to n terms of the series 11, 23, 59, 167 ………is

(A) $3^{n+1}+5n-3$ (B) $3^{n+1}+5n+3$ (C) 3^n+5n-3 (D) None

63. The sum of n terms of the series $1/(4.9)+1/(9.14)+1/(14.19)+1/(19.24)+........$is

(A) $(n/4)(5n+4)^{-1}$ (B) $(n/4)(5n+4)$ (C) $(n/4)(5n-4)^{-1}$ (D) None

64. The sum of n terms of the series 1 + 3 + 5 + ………. Is

(A) n^2 (B) $2n^2$ (C) $n^2/2$ (D) None

65. The sum of n terms of the series 2 + 6 + 10 + ……. is

(A) $2n^2$ (B) n^2 (C) $n^2/2$ (D) $4n^2$

66. The sum of n terms of the series 1.2 + 2.3 + 3.4 + ……. Is

(A) $(n/3)(n+1)(n+2)$ (B) $(n/2)(n+1)(n+2)$ (C) $(n/3)(n+1)(n-2)$(D) None

67. The sum of n terms of the series 1.2.3 + 2.3.4 + 3.4.5 + …….is

(A) $(n/4)(n+1)(n+2)(n+3)$ (B) $(n/3)(n+1)(n+2)(n+3)$

(C) $(n/2)(n+1)(n+2)(n+3)$ (D) None

68. The sum of n terms of the series $1.2+3.2^2+5.2^3+7.2^4+$....... is

(A) $(n–1)2^{n+2}–2^{n+1}+6$ (B) $(n+1)2^{n+2}–2^{n+1}+6$ (C) $(n–1)2^{n+2}–2^{n+1}–6$ (D) None

69. The sum of n terms of the series 1/(3.8)+1/(8.13)+1/(13.18)+...... is

(A) $(n/3)(5n+3)^{–1}$ (B) $(n/2)(5n+3)^{–1}$ (C) $(n/2)(5n–3)^{–1}$ (D) None

70. The sum of n terms of the series 1/1+1/(1+2)+1/(1+2+3)+..... is

(A) $2n(n+1)^{–1}$ (B) $n(n+1)$ (C) $2n(n–1)^{–1}$ (D) None

71. The sum of n terms of the series $2^2+5^2+8^2+$........ is

(A) $(n/2)(6n^2+3n–1)$ (B) $(n/2)(6n^2–3n–1)$

(C) $(n/2)(6n^2+3n+1)$ (D) None

72. The sum of n terms of the series $1^2+3^2+5^2+$........ is

(A) $\frac{n}{3}(4n^2-1)$ (B) $n^2(2n^2+1)$ (C) $n(2n–1)$ (D) $n(2n+1)$

73. The sum of n terms of the series 1.4 + 3.7 + 5.10 + …… is

(A) $(n/2)(4n^2+5\quad 1)$ (B) $(n/2)(5n^2+4n–1)$

(C) $(n/2)(4n^2+5n+1)$ (D) None

74. The sum of n terms of the series $2.3^2+5.4^2+8.5^2+$........ is

(A) $(n/12)(9n^3+62n^2+123n+22)$ (B) $(n/12)(9n^3-62n^2+123n-22)$

(C) $(n/6)(9n^3+62n^2+123n+22)$ (D) None

75. The sum of n terms of the series 1 + (1 + 3) + (1 + 3 + 5) + ……. is

(A) $(n/6)(n+1)(2n+1)$ (B) $(n/6)(n+1)(n+2)$ (C) $(n/3)(n+1)(2n+1)$ (D) None

76. The sum of n terms of the series $1^2+(1^2+2^2)+(1^2+2^2+3^2)+$........ is

(A) $(n/12)(n+1)^2(n+2)$ (B) $(n/12)(n-1)^2(n+2)$ (C) $(n/12)(n^2-1)(n+2)$ (D) None

77. The sum of n terms of the series $1+(1+1/3)+(1+1/3+1/3^2)+........$ is

(A) $(3/2)(1-3^{-n})$ (B) $(3/2)[n-(1/2)(1-3^{-n})]$ (C) Both (D) None

78. The sum of n terms of the series $n.1+(n-1).2+(n-2).3+$ .... is

(A) $(n/6)(n+1)(n+2)$ (B) $(n/3)(n+1)(n+2)$ (C) $(n/2)(n+1)(n+2)$ (D) None

79. The sum of n terms of the series 1 + 5 + 12 + 22 + ….. is

(A) $(n^2/2)(n+1)$ (B) $n^2(n+1)$ (C) $(n^2/2)(n-1)$ (D) None

80. The sum of n terms of the series 4 + 14 + 30 + 52 + 80 + …… is

(A) $n(n+1)^2$ (B) $n(n-1)^2$ (C) $n(n^2-1)$ (D) None

81. The sum of n terms of the series 3 + 6 + 11 + 20 + 37 + …….. is

(A) $2^{n+1}+(n/2)(n+1)-2$ (B) $2^{n+1}+(n/2)(n+1)-1$ (C) $2^{n+1}+(n/2)(n-1)-2$ (D) None

82. The n^{th} terms of the series is 1/(4.7) + 1/(7.10) + 1/(10.13) + ……. is

(A) $(1/3)[(3n+1)^{-1}-(3n+4)^{-1}]$ (B) $(1/3)[(3n-1)^{-1}-(3n+4)^{-1}]$

(C) $(1/3)[(3n+1)^{-1}-(3n-4)^{-1}]$ (D) None

83. In question No.(82) the sum of the series upto µ is

(A) $(n/4)(3n+4)^{-1}$ (B) $(n/4)(3n-4)^{-1}$ (C) $(n/2)(3n+4)^{-1}$ (D) None

84. The sum of n terms of the series $1^2/1+(1^2+2^2)/(1+2)+(1^2+2^2+3^2)/(1+2+3)+....$ is

(A) $(n/3)(n+2)$ (B) $(n/3)(n+1)$ (C) $(n/3)(n+3)$ (D) None

85. The sum of n terms of the series $1^3/1+(1^3+2^3)/2+(1^3+2^3+3^3)/3+....$ is

(A) $(n/48)(n+1)(n+2)(3n+5)$ (B) $(n/24)(n+1)(n+2)(3n+5)$

(C) $(n/48)(n+1)(n+2)(5n+3)$ (D) None

86. The value of $n^2++2n[1+2+3+.....+(n-1)]$ is

(A) n^3 (B) n^2 (C) n (D) None

87. $2^{4n}-1$ is divisible by

(A) 15 (B) 4 (C) 6 (D) 64

88. $3^n - 2n - 1$ is divisible by

(A) 15 (B) 4 (C) 6 (D) 64

89. $n(n-1)(2n-1)$ is divisible by

(A) 15 (B) 4 (C) 6 (D) 64

90. $7^{2n} + 16n - 1$ is divisible by

(A) 15 (B) 4 (C) 6 (D) 64

91. The sum of n terms of the series whose n^{th} term $3n^2 + 2n$ is is given by

(A) $(n/2)(n+1)(2n+3)$ (B) $(n/2)(n+1)(3n+2)$

(C) $(n/2)(n+1)(3n-2)$ (D) $(n/2)(n+1)(2n-3)$

92. The sum of n terms of the series whose n^{th} term $n.2^n$ is is given by

(A) $(n-1)2^{n+1}+2$ (B) $(n+1)2^{n+1}+2$ (C) $(n-1)2^n+2$ (D) None

93. The sum of n terms of the series whose n^{th} term $5.3^{n+1}+2n$ is is given by

(A) $(5/2)(3^{n+2}-9)+n(n+1)$ (B) $(2/5)(3^{n+2}-9)+n(n+1)$

(C) $(5/2)(3^{n+2}+9)+n(n+1)$ (D) None

94. If the third term of a G.P. is the square of the first and the fifth term is 64 the series would be ________.

(A) 4 + 8 + 16 + 32 + …. (B) 4 – 8 + 16 – 32 + ……..

(C) both (D) None

95. Three numbers whose sum is 15 are in A.P. but if they are added by 1, 4, 19 respectively they are in G.P. The numbers are _______.

(A) 2, 5, 8 (B) 26, 5, –16 (C) Both (D) None

96. If a, b, c are the p^{th}, q^{th} and r^{th} terms of a G.P. respectively the value of $a^{q-r}.b^{r-p}.c^{p-q}$ is ________

(A) 0 (B) 1 (C) –1 (D) None

97. If a, b, c are in A.P. and x, y, z in G.P. then the value of $x^{b-c}.y^{c-a}.z^{a-b}$ is ________

(A) 0 (B) 1 (C) –1 (D) None

98. If *a, b, c* are in A.P. and *x, y, z* in G.P. then the value of $(x^b.y^c.z^a)\div(x^c.y^a.z^b)$ is ____

(A) 0 (B) 1 (C) –1 (D) None

99. The sum of *n* terms of the series 7 + 77 + 777 + …… is

(A) $(7/9)[(1/9)(10^{n+1}-10)-n]$ (B) $(9/10)[(1/9)(10^{n+1}-10)-n]$

(C) $(10/9)[(1/9)(10^{n+1}-10)-n]$ (D) None

100. The least value of n for which the sum of n terms of the series $1 + 3 + 3^2 +$ …… is greater than 7000 is ______.

(A) 9 (B) 10 (C) 8 (D) 7

101. If '*S*' be the sum, '*P*' the product and '*R*' the sum of the reciprocals of *n* terms in a G.P. then '*P*' is the _______ of S^n and R^{-n}.

(A) Arithmetic Mean (B) Geometric Mean (C) Harmonic Mean (D) None

102. Sum upto ∞ of the series $8+4\sqrt{2}+4+.....$ is

(A) $8(2+\sqrt{2})$ (B) $8(2-\sqrt{2})$ (C) $4(2+\sqrt{2})$ (D) $4(2-\sqrt{2})$

103. Sum upto ∞ of the series $1/2+1/3^2+1/2^3+1/3^4+1/2^5+1/3^6+.....$ is

(A) 19/24 (B) 24/19 (C) 5/24 (D) None

104. If $1+a+a^2+.......a=x$ and $1+b+b^2+.......a=y$ then $1+ab+a^2b^2+......a$ is given by __________.

(A) $(xy)/(x+y-1)$ (B) $(xy)/(x-y-1)$ (C) $(xy)/(x+y+1)$ (D) None

105. If the sum of three numbers in G.P. is 35 and their product is 1000 the numbers are ____.

(A) 20, 10, 5 (B) 5, 10, 20 (C) both (D) None

106. If the sum of three numbers in G.P. is 21 and the sum of their squares is 189 the numbers are ____.

(A) 3, 6, 12 (B) 12, 6, 3 (C) both (D) None

107. If *a, b, c* are in G.P. then the value of $a(b^2+c^2)-c(a^2+b^2)$ is ____

(A) 0 (B) 1 (C) –1 (D) None

108. If *a, b, c, d* are in G.P. then the value of $b(ab-cd)-(c+a)(b^2-c^2)$ is ____

(A) 0 (B) 1 (C) –1 (D) None

109. If *a, b, c, d* are in G.P. then the value of $(ab+bc+cd)^2-(a^2+b^2+c^2)(b^2+c^2+d^2)$ is _________.

(A) 0 (B) 1 (C) –1 (D) None

110. If *a, b, c, d* are in G.P. then $a+b, b+c, c+d$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None

111. If *a, b, c* are in G.P. then $a^2+b^2, ab+bc, b^2+c^2$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None

112. If *a, b, x, y, z* are positive numbers such that *a, x, b* are in A.P. and *a, y, b* are in G.P. and $z=(2ab)/(a+b)$ then

(A) *x y z* are in G.P. (B) $x \geq y \geq z$ (C) both (D) None

113. If *a, b, c* are in G.P. then the value of $(a-b+c)(a+b+c)^2-(a+b+c)(a^2+b^2+c^2)$ is given by

(A) 0 (B) 1 (C) –1 (D) None

114. If *a, b, c* are in G.P. then the value of $a(b^2+c^2)-c(a^2+b^2)$ is given by

(A) 0 (B) 1 (C) –1 (D) None

115. If *a, b, c* are in G.P. then the value of $a^2b^2c^2(a^{-3}+b^{-3}+c^{-3})-(a^3+b^3+c^3)$ is given by

(A) 0 (B) 1 (C) –1 (D) None

116. If *a, b, c, d* are in G.P. then $(a-b)^2, (b-c)^2, (c-d)^2$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None

117. If *a b c d* are in G.P. then the value of $(b-c)^2+(c-a)^2+(d-b)^2-(a-d)^2$ is given by

(A) 0 (B) 1 (C) –1 (D) None

118. If $(a-b), (b-c), (c-a)$ are in G.P. then the value of $(a+b+c)^2-3(ab+bc+ca)$ is given by

(A) 0 (B) 1 (C) –1 (D) None

119. If $a^{1/x}=b^{1/y}=c^{1/z}$ and *a, b, c* are in G.P. then *x, y, z* are in

(A) A.P. (B) G.P. (C) H.P. (D) None

120. If x = a + a/r + a/r² + ... α, y = b – b/r + b/r² – ..... α, and z = c + c/r² + c/r⁴ + ... α, then the value of $\frac{xy}{z}-\frac{ab}{c}$ is

(A) 0 (B) 1 (C) –1 (D) None

121. If *a, b, c* are in A.P. *a, x, b* are in G.P. and *b, y, c* are in G.P then x^2, b^2, y^2 are in

(A) A.P. (B) G.P. (C) H.P. (D) None

122. If a, b-a, c-a are in G.P. and a=b/3=c/5 then a, b, c are in

(A) A.P. (B) G.P. (C) H.P. (D) None

123. If a, b, (c+1) are in G.P. and a = $(b-c)^2$ then a, b, c are in

(A) A.P. (B) G.P. (C) H.P. (D) None

124. If $S_1, S_2, S_3, \ldots\ldots S_n$ are the sums of infinite G.P.s whose first terms are 1, 2, 3 …..n and whose common ratios are 1/2, 1/3, ……1/(*n*+1) then the value of $S_1+S_2+S_3+\ldots\ldots S_n$ is

(A) (n/2) (n+3) (B) (n/2) (n+2) (C) (n/2) (n+1) (D) $n^2/2$

125. The G.P. whose 3rd and 6th terms are 1, –1/8 respectively is

(A) 4, –2, 1 ….. (B) 4, 2, 1 ……. (C) 4, –1, 1/4 …….. (D) None

126. In a G.P. if the $(p+q)^{th}$ term is *m* and the $(p-q)^{th}$ term is *n* then the p^{th} term is_________.

(A) $(mn)^{1/2}$ (B) mn (C) (m+n) (D) (m-n)

127. The sum of *n* terms of the series is $1/\sqrt{3}+1+3/\sqrt{3}+\ldots$

(A) $(1/6)(3+\sqrt{3})(3^{n/2}-1)$, (B) $(1/6)(\sqrt{3}+1)(3^{n/2}-1)$,

(C) $(1/6)(3+\sqrt{3})(3^{n/2}+1)$, (D) None

128. The sum of *n* terms of the series 5/2 – 1 + 2/5 – …… is

(A) $(1/14)(5^n+2^n)/5^{n-2}$ (B) $(1/14)(5^n-2^n)/5^{n-2}$ (C) both (D) None

129. The sum of *n* terms of the series 0.3 + 0.03 + 0.003 + …….. is

(A) $(1/3)(1-1/10^n)$ (B) $(1/3)(1+1/10^n)$ (C) both (D) None

130. The sum of first eight terms of G.P. is five times the sum of the first four terms. The common ratio is _______.

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) both (D) None

131. If the sum of *n* terms of a G.P. with first term 1 and common ratio 1/2 is 1+127/128, the value of *n* is _______.

(A) 8 (B) 5 (C) 3 (D) None

132. If the sum of n terms of a G.P. with last term 128 and common ratio 2 is 255, the value of n is _________.

(A) 8 (B) 5 (C) 3 (D) None

133. How many terms of the G.P. 1, 4, 16 …. are to be taken to have their sum 341?

(A) 8 (B) 5 (C) 3 (D) None

134. The sum of n terms of the series 5 + 55 + 555 + …….. is

(A) $(50/81)\,(10^n-1)-(5/9)n$ (B) $(50/81)\,(10^n+1)-(5/9)n$

(C) $(50/81)\,(10^n+1)+(5/9)n$ (D) None

135. The sum of n terms of the series 0.5 + 0.55 + 0.555 + ………. is

(A) $(5/9)n-(5/81)(1-10^{-n})$ (B) $(5/9)n+(5/81)(1-10^{-n})$

(C) $(5/9)n+(5/81)(1+10^{-n})$ (D) None

136. The sum of n terms of the series $1.03+1.03^2+1.03^3+......$ is

(A) $(103/3)(1.03^n-1)$ (B) $(103/3)(1.03^n+1)$ (C) $(103/3)(1.03^{n+1}-1)$ (D) None

137. The sum upto infinity of the series 1/2 + 1/6 + 1/18 + …… is

(A) 3/4 (B) 1/4 (C) 1/2 (D) None

138. The sum upto infinity of the series 4 + 0.8 + 0.16 + …… is

(A) 5 (B) 10 (C) 8 (D) None

139. The sum upto infinity of the series $\sqrt{2}+1/\sqrt{2}+1/(2\sqrt{2})+.......$ is

(A) $2\sqrt{2}$ (B) 2 (C) 4 (D) None

140. The sum upto infinity of the series 2/3 + 5/9 + 2/27 + 5/81 + ……. is

(A) 11/8 (B) 8/11 (C) 3/11 (D) None

141. The sum upto infinity of the series $(\sqrt{2}+1)+1+(\sqrt{2}-1)+......$ is

(A) $(1/2)(4+3\sqrt{2})$ (B) $(1/2)(4-3\sqrt{2})$ (C) $4+3\sqrt{2}$ (D) None

142. The sum upto infinity of the series $(1+2^{-2})+(2^{-1}+2^{-4})+(2^{-2}+2^{-6})+.......$ is

(A) 7/3 (B) 3/7 (C) 4/7 (D) None

143. The sum upto infinity of the series $4/7-5/7^2+4/7^3-5/7^4+$ .... is

(A) 23/48 (B) 25/48 (C) 1/2 (D) None

144. If the sum of infinite terms in a G.P. is 2 and the sum of their squares is 4/3 the series is

(A) 1, 1/2, 1/4 …… (B) 1, –1/2, 1/4 ……. (C) –1, –1/2, –1/4 …. (D) None

145. The infinite G.P. series with first term 1/4 and sum 1/3 is

(A) 1/4, 1/16, 1/64 … (B) 1/4, –1/16, 1/64 …(C) 1/4, 1/8, 1/16 …. (D) None

146. If the first term of a G.P. exceeds the second term by 2 and the sum to infinity is 50 the series is __________.

(A) 10, 8, 32/5 … (B) 10, 8, 5/2 … (C) 10, 10/3, 10/9 …. (D) None

147. Three numbers in G.P. with their sum 130 and their product 27000 are _________.

(A) 10, 30, 90 … (B) 90, 30, 10 … (C) both (D) None

148. Three numbers in G.P. with their sum 13/3 and sum of their squares 91/9 are ____.

(A) 1/3, 1, 3 (B) 3, 1, 1/3 (C) both (D) None

149. Find five numbers in G.P. such that their product is 32 and the product of the last two is 108.

(A) 2/9, 2/3, 2, 6, 18 (B) 18, 6, 2, 2/3, 2/9 (C) both (D) None

150. If the continued product of three numbers in G.P. is 27 and the sum of their products in pairs is 39 the numbers are _________.

(A) 1, 3, 9 (B) 9, 3, 1 (C) both (D) None

151. The numbers *x,* 8, *y* are in G.P. and the numbers *x, y,* –8 are in A.P. The values of *x, y* are ___________.

(A) 16, 4 (B) 4, 16 (C) both (D) None

ANSWERS

1)	C	31)	A	61)	A	91)	A	121)	A
2)	A	32)	A	62)	A	92)	A	122)	A
3)	B	33)	A	63)	A	93)	A	123)	A
4)	A	34)	B	64)	A	94)	C	124)	A
5)	A	35)	B	65)	A	95)	C	125)	A
6)	B	36)	A	66)	A	96)	B	126)	A
7)	C	37)	B	67)	A	97)	B	127)	A
8)	C	38)	C	68)	A	98)	B	128)	C
9)	A	39)	D	69)	A	99)	A	129)	A
10)	A	40)	D	70)	A	100)	C	130)	C
11)	B	41)	A	71)	A	101)	B	131)	A
12)	B	42)	A	72)	A	102)	A	132)	A
13)	A	43)	D	73)	A	103)	A	133)	B
14)	C	44)	D	74)	A	104)	A	134)	A
15)	B	45)	C	75)	A	105)	C	135)	A
16)	A	46)	A	76)	A	106)	C	136)	A
17)	A	47)	B	77)	B	107)	A	137)	A
18)	A	48)	C	78)	A	108)	A	138)	A
19)	A	49)	A	79)	A	109)	A	139)	A
20)	C	50)	B	80)	A	110)	B	140)	A
21)	B	51)	A	81)	A	111)	B	141)	A
22)	A	52)	D	82)	A	112)	C	142)	A
23)	A	53)	B	83)	A	113)	A	143)	A
24)	A	54)	B	84)	A	114)	A	144)	A
25)	C	55)	A	85)	A	115)	A	145)	A
26)	A	56)	A	86)	A	116)	B	146)	A
27)	C	57)	D	87)	A	117)	A	147)	C
28)	C	58)	A	88)	B	118)	A	148)	C
29)	A	59)	A	89)	C	119)	A	149)	A
30)	C	60)	A	90)	D	120)	A	150)	C
151)	A								